

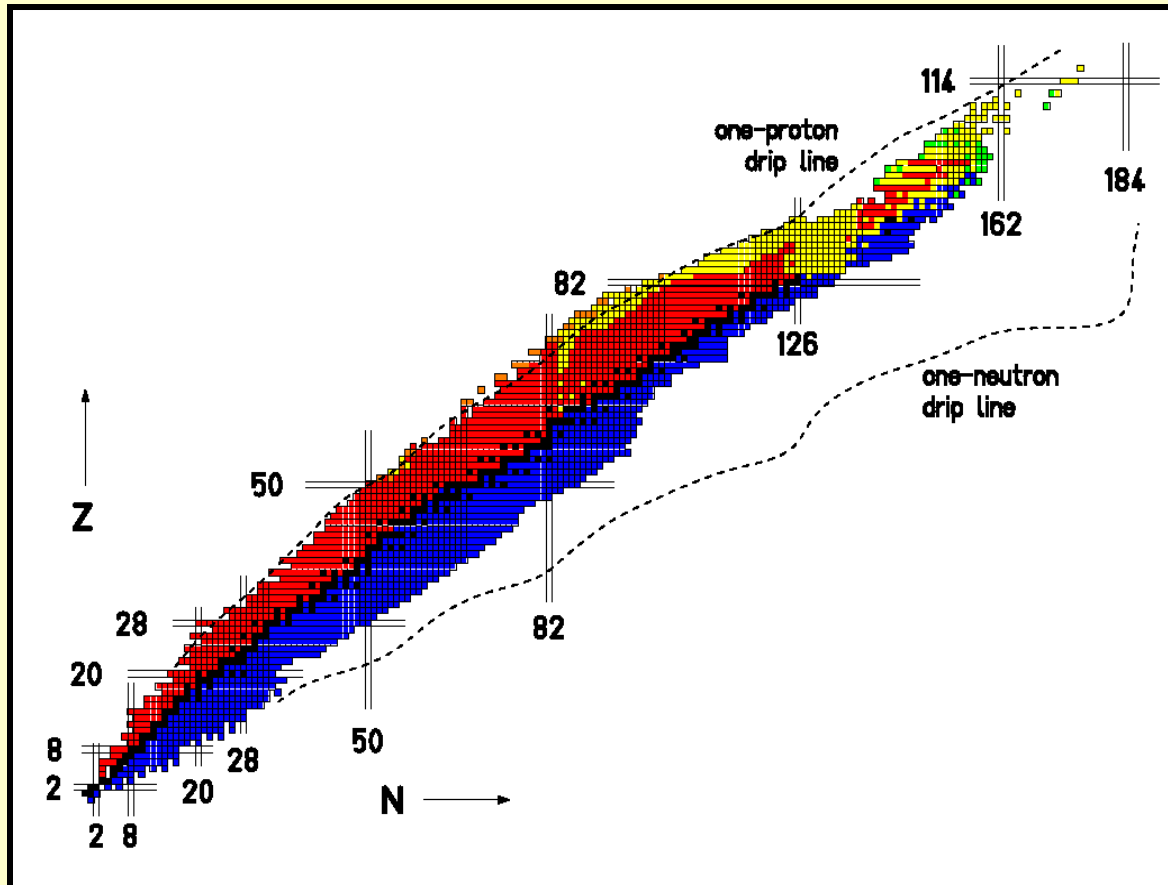
Nuclear structure - University of Ghent

- Nuclear shape coexistence and shape phase transitions in atomic nuclei
- The geometrical Bohr-Mottelson model: new analytic solutions and an algebraic Cartan-Weyl perspective
- The changing mean-field in exotic nuclei: a shell-model point of view

Nuclear structure - Topic 1

Nuclear shape coexistence and shape phase transitions in atomic nuclei

K.Heyde, V.Hellemans (Postdoc. Univ. Notre-Dame), S.De Baerdemacker (Postdoc FWO), J.E. Garcia-Ramos (Univ. Huelva, Spain) (in coll. with P. Van Isacker (GANIL) and J.L. Wood (Georgia Tech., Atlanta))

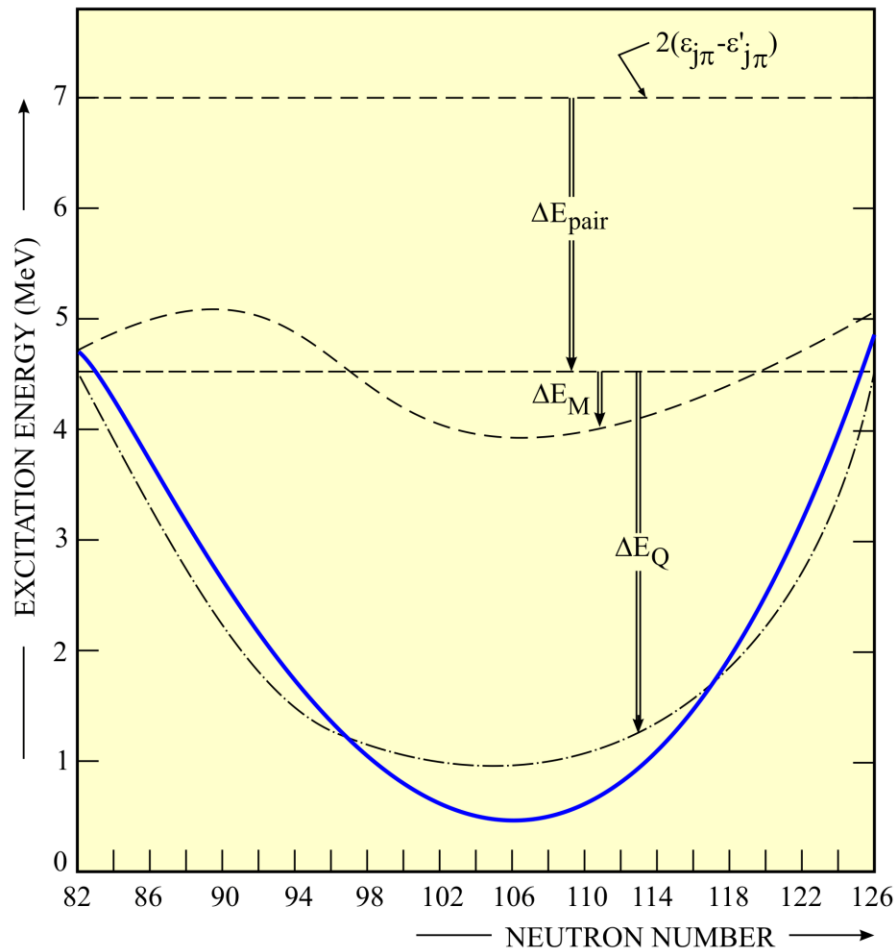


Aim : explore conditions in the nuclear landscape for coexistence of various phases.

Importance of the interplay between stabilizing effect of spherical closed shells versus the residual interaction energy for certain distributions of valence protons and neutrons or excitations across 'closed' shells

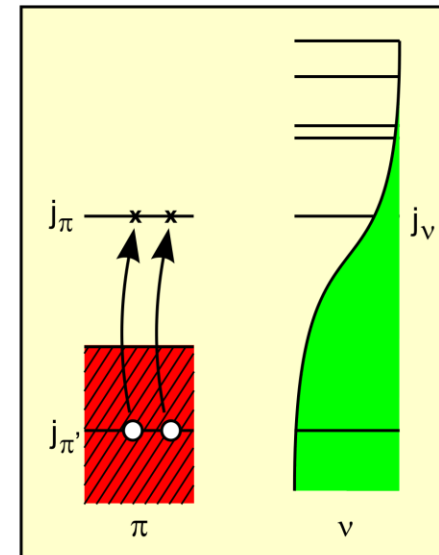
B. HEAVY-NUCLEI WITH NEUTRON EXCESS

Huge model space cannot be extended, including p-h excitations
 \Rightarrow symmetry as guide to truncation.



Consider lowest 2p-2h excitations across closed shells.

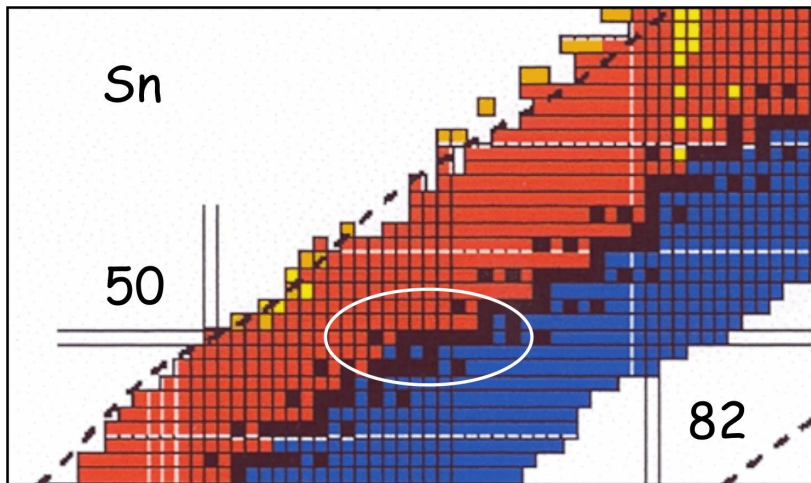
$Q_\pi \cdot Q_\nu$ quadrupole-quadrupole force



$$E_{\text{intr.}}(0^+) \simeq 2(\epsilon_p - \epsilon_h) + \Delta E_{\text{pair.}} + \Delta E_{\text{mon.}} + \Delta E_{\text{quad.}}$$

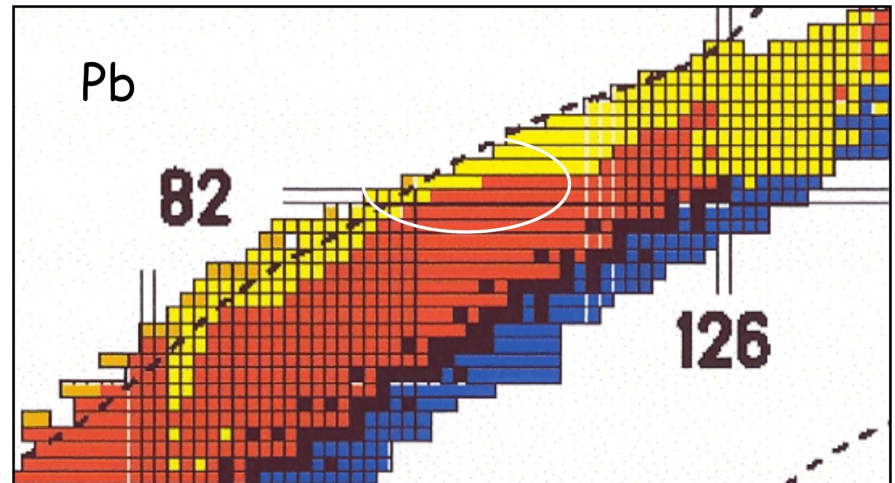
$$\Delta E_{\text{quad.}} \simeq 2\kappa N_\nu \cdot \Delta N_\pi$$

Conditions: closed shell for protons-large 'open' shell for neutrons



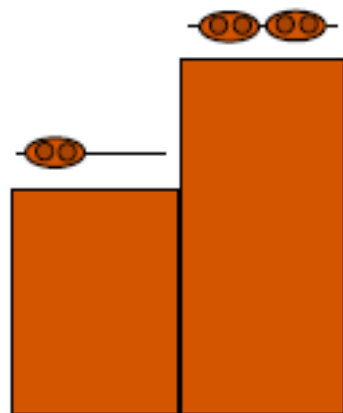
Zone of interest overlays
with stable Sn nuclei

How do the data look: the Pb region.



Zone of interest for very
neutron-deficient Pb nuclei

A single configuration

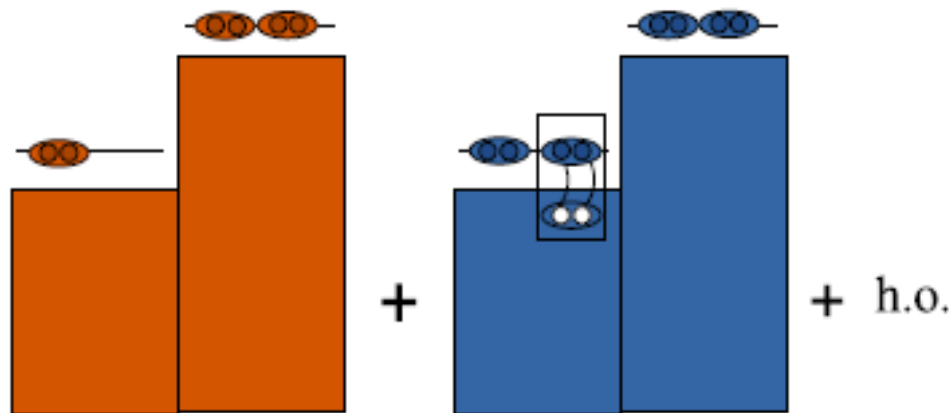


The essential physics of the model is contained in the consistent-Q Hamiltonian

$$\hat{H}_{\text{cqf}}^N = \varepsilon \hat{n}_d + \kappa \hat{Q}(\chi) \cdot \hat{Q}(\chi)$$

- N is the number of bosons
- ε , κ , and χ are parameters

Configuration mixing



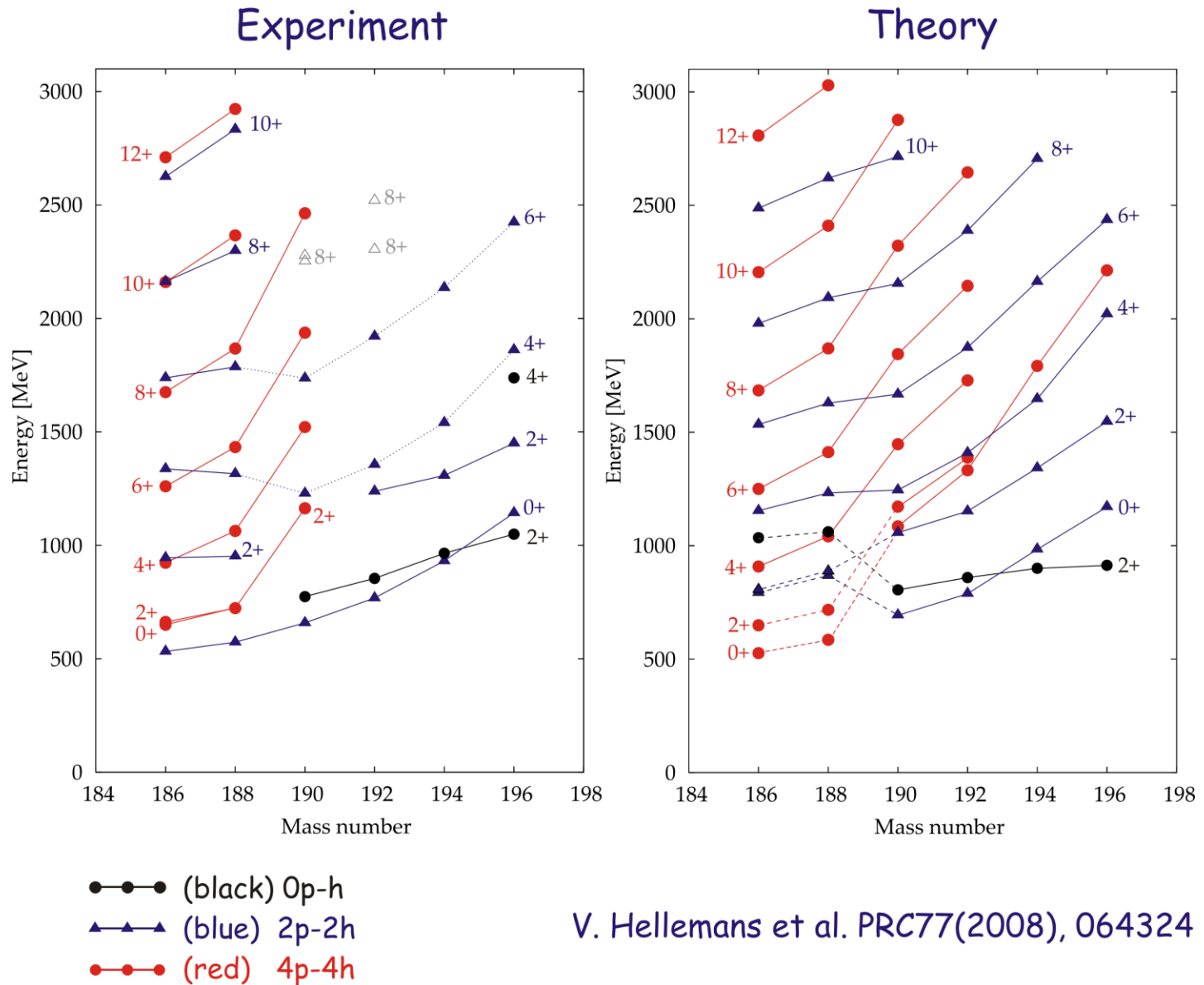
The Hamiltonian can be extended to include particle-hole configurations

$$\begin{aligned} \hat{H} = & \hat{P}_N^\dagger \hat{H}_{\text{cqf}}^N \hat{P}_N \\ & + \hat{P}_{N+2}^\dagger \left(\hat{H}_{\text{cqf}}^{N+2} + \Delta^{N+2} \right) \hat{P}_{N+2} \\ & + \hat{V}_{\text{mix}}^{N, N+2} + \text{h.o.} \end{aligned}$$

Δ^{N+2} takes corrected excitation energy of 2p-2h configuration into account

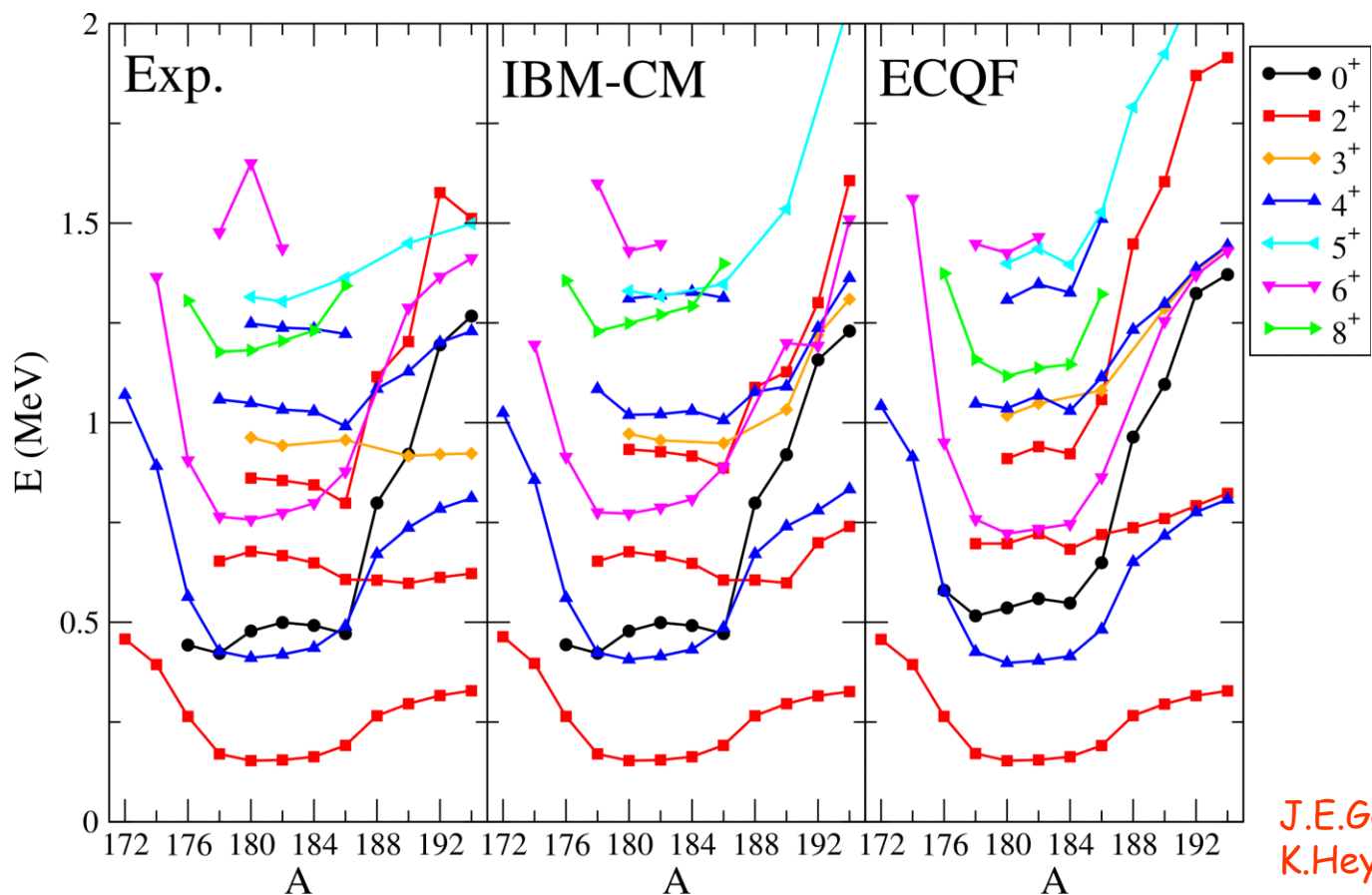
Applied to Pb nuclei: Hellemans et al., PRC71(2005);034308 and PRC77(2008),064324;
Pakarinen et al., PRC75(2007),014302.

Neutron-deficient Pb nuclei: systematics of bands.



V. Hellemans et al. PRC77(2008), 064324

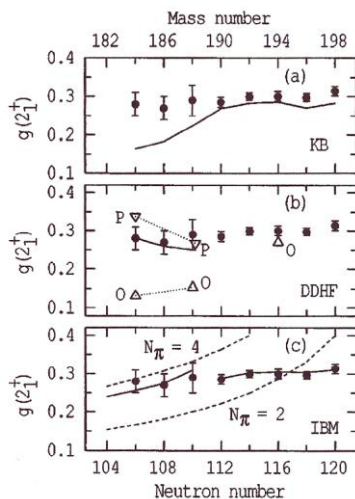
Comparison between (i) exp. data, (ii) IBM-configuration mixing ($[N] \oplus [N+2]$), and, (iii) ECQF calculation, using small model space ($[N]$ space, only).
 $E_x \approx 1.5$ MeV.



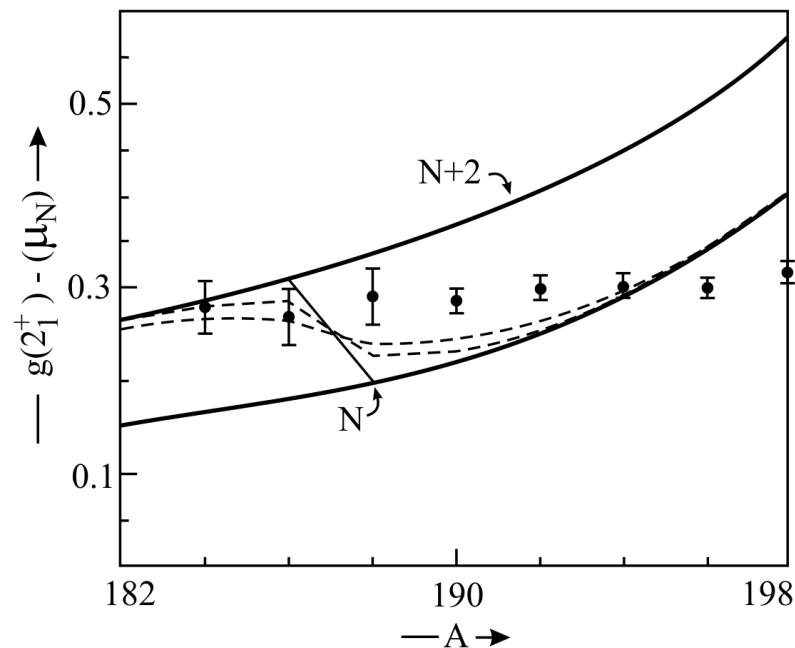
Pt-nuclei

J.E.Garcia-Ramos and
K.Heyde, NPA825(2009),39

$g(2^+)$ factor and in even-even Pt nuclei



Stuchbery et al., PRL76(1996),2246

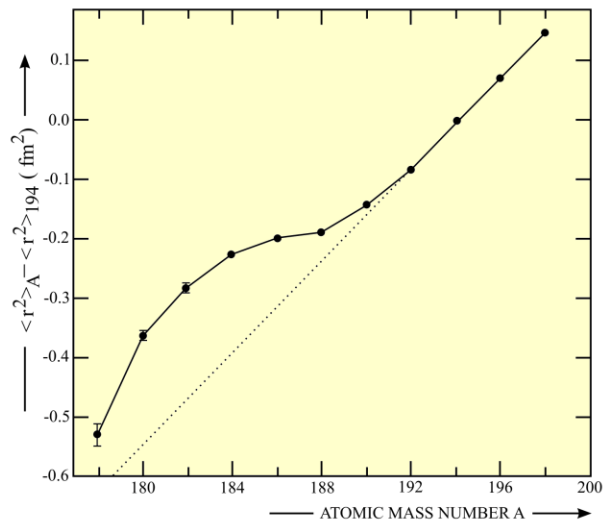


Harder et al., PLB405(1997),25

Need for the configurations with $N+2$ bosons

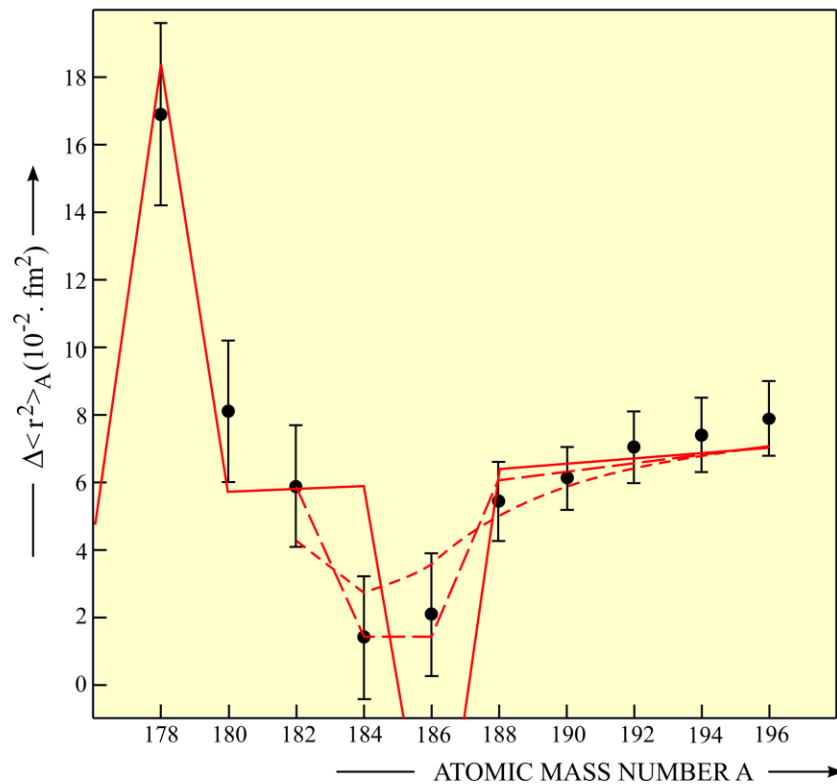
ISOTOPIC SHIFTS IN THE P+ NUCLEI

$$\langle r^2 \rangle_A - \langle r^2 \rangle_{194} \text{ (fm}^2\text{)}$$



Hilberath et al., Z. Phys. A342('92), 1
Le Blanc et al.,PRC60('99), 054310

$$\Delta \langle r^2 \rangle_A \equiv \langle r^2 \rangle_{A+2} - \langle r^2 \rangle_A$$



IBM-CM schematic
model parameters

Harder et al.,
PLB405('97), 25

Research Topics - present and future projects

Study of shape coexistence in the Pb region - how are intruder configurations evolving away from the $Z=82$ closed shell. This is in close collaborations with experimentalists (Leuven, Jyväskylä, ISOLDE-CERN,..): work in progress on Pt,Hg nuclei.

Study of shape transition in the $N=90$ region and in the Zr,Sr region approaching the $N=60$ region of deformation. This is in close collaboration with J.Wood et al. (Georgia Tech., Atlanta)

Exploring ways to connect the description of intruder excitations near closed shells, with the onset of deformation using group-theoretical methods (IBM, phase transitions,..)
(see NPA825(2009),39)

Nuclear structure - Topic 2

The geometrical Bohr-Mottelson model: new analytic solutions and an algebraic Cartan-Weyl perspective

S. De Baerdemacker (Postdoc FWO), presently at Univ. Toronto (in collaboration with D.Rowe, T.Welsh).

What's so geometrical about the geometrical model?



- Macroscopically, the atomic nucleus can be compared to a charged liquid drop.
- Deviations from the sphere are developed in **multipole** orders.

Radius

$$R(\theta, \phi) = R_0 \left[1 + \sum_l \alpha_l \cdot Y^l(\theta, \phi) \right]$$

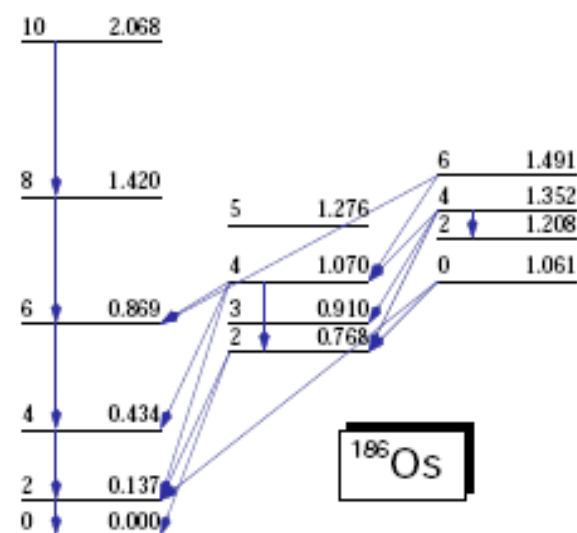
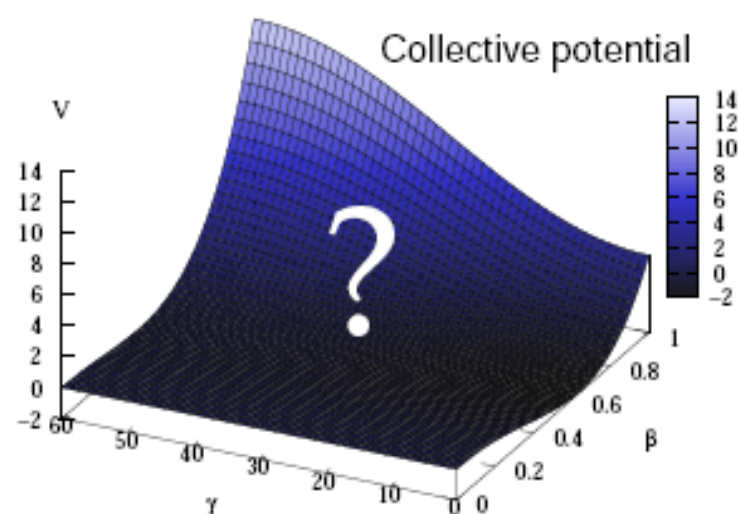
- $\alpha_{l\mu} ::$ collective coordinates
 $Y_m^l(\theta, \phi) ::$ spherical harmonics

input/output of the geometrical model

The Bohr Hamiltonian

$$\hat{H} = \hat{T} + V(\alpha)$$

$$V(\alpha) = c_2(\alpha \cdot \alpha) + c_3([\alpha\alpha]^2 \cdot \alpha) + c_4(\alpha \cdot \alpha)^2 + c_5([\alpha\alpha]^2 \cdot \alpha)(\alpha \cdot \alpha) + \mathcal{O}(6)$$



- Search for potentials to describe experimental data (Energy spectra, $B(E2)$, quadrupole moments,...).
- Diagonalise the Hamiltonian, so find a **matrix representations** within a suitable basis.

Why $SU(1,1) \times O(5)$?

The Hamiltonian

$$\hat{H} = \frac{1}{2B_2} \pi \cdot \pi + B_3 \pi \cdot [\alpha \pi]^2 + c_2(\alpha \cdot \alpha) + c_3([\alpha \alpha]^2 \cdot \alpha) + c_4(\alpha \cdot \alpha)^2 \\ + c_5([\alpha \alpha]^2 \cdot \alpha)(\alpha \cdot \alpha) + c_6(\alpha \cdot \alpha)^3 + d_6([\alpha \alpha]^2 \cdot \alpha)^2 + \dots$$

- $SU(1,1)$ describes the β excitations

$$\alpha \cdot \alpha = \beta^2$$

- $SO(5)$ describes the substructure in γ and angular momentum

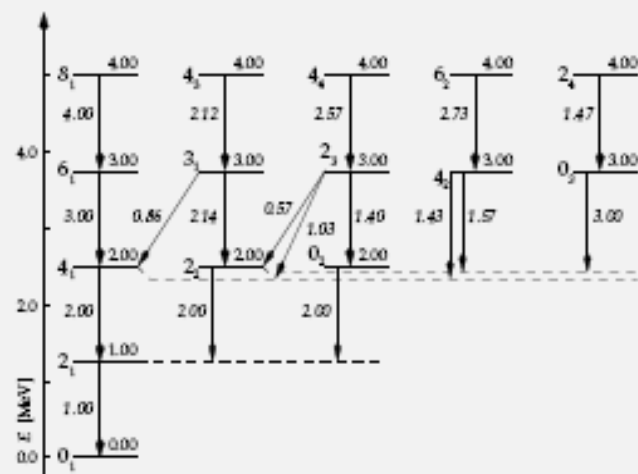
$$[\alpha \alpha]^2 \cdot \alpha = -\sqrt{\frac{2}{7}} \beta^3 \cos 3\gamma, T(E2) = \alpha$$

- Basis? $|n\lambda\rangle |v, \nu_\Delta, L, M\rangle$

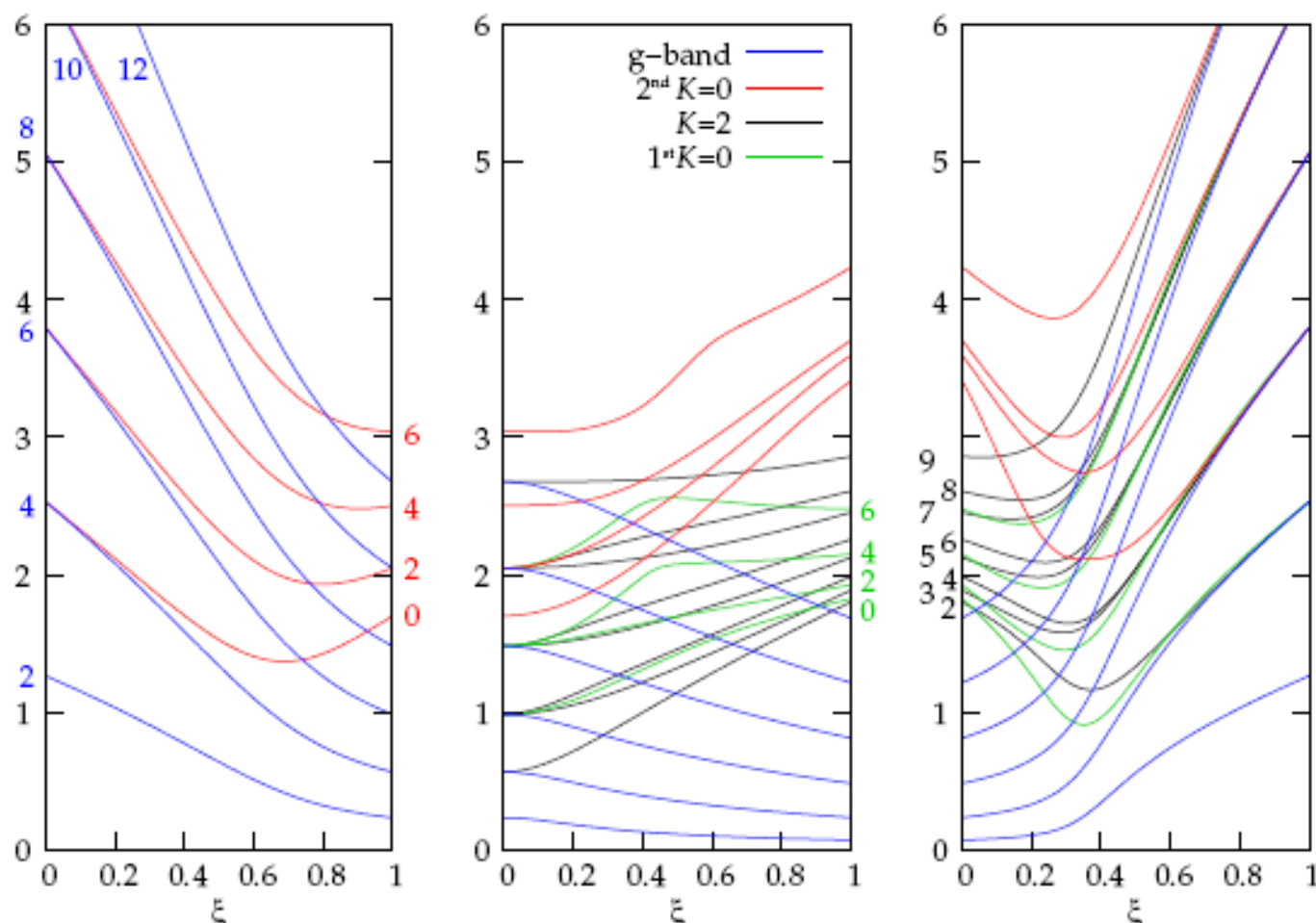
$$\underbrace{SU(1,1)}_{(n,\lambda)} \times \underbrace{SO(5)}_v \supset \underbrace{SO(3)}_{\nu_\Delta} \supset \underbrace{SO(2)}_M$$

Example: 5D H.O.

$$\hat{H} = \frac{1}{2B_2} \pi \cdot \pi + c_2(\alpha \cdot \alpha)$$



From one limit to the other: energy spectrum



vibrator $\rightarrow \gamma$ -independent rotor \rightarrow axial rotor \rightarrow vibrator

Research Topics - present and future projects

Study of the spectral properties of an algebraic tractable Hamiltonian using a Cartan-Weyl basis and the $SU(1,1) \times SO(5)$ structure of quadrupole motion.

The vibrational, γ -independent rotational and axially deformed structures, as well as transitional regions are discussed

Recent Ph.D. Thesis of S. De Baerdemacker

Future developments: use more general potentials to cover shape coexisting structures and phase transitions, in collaboration with the group at the University of Toronto (D.Rowe and co-workers)

Nuclear structure - Topic 3

The changing mean-field in exotic nuclei: a shell-model point of view

N.Smirnova (CENG-Bordeaux), K.Heyde

The Monopole Hamiltonian

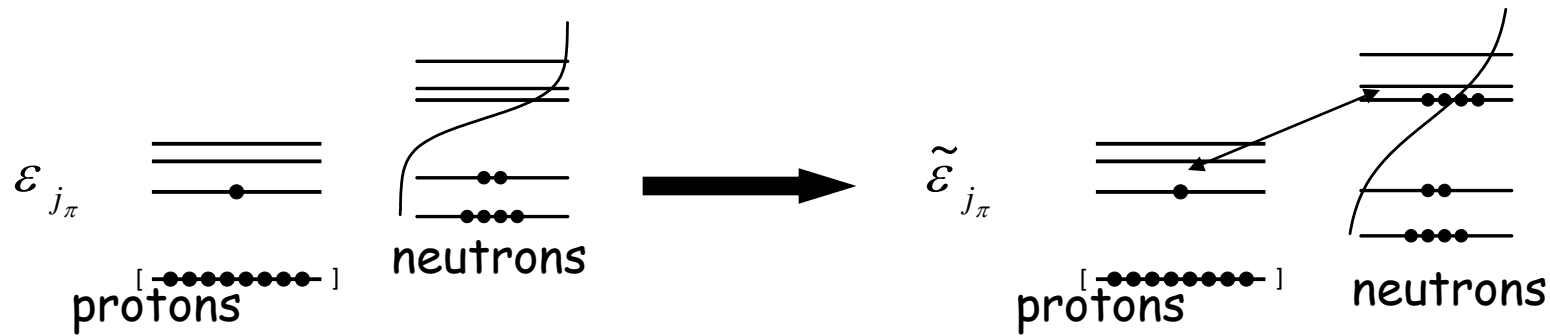
$$H = \sum_{j_\pi} \varepsilon_{j_\pi} \hat{n}_{j_\pi} + \sum_{j_\nu} \varepsilon_{j_\nu} \hat{n}_{j_\nu} + V_{\pi\pi} + V_{\nu\nu} + V_{\pi\nu}$$

- recoupling $V_{\pi\nu}$ from pp to ph basis \rightarrow multipole expansion
- keeping only monopole term $\lambda = 0$ of

$$H_{\lambda=0} = \sum_{j_\pi} \varepsilon_{j_\pi} \hat{n}_{j_\pi} + \sum_{j_\nu} \varepsilon_{j_\nu} \hat{n}_{j_\nu} + \sum_{j_\pi, j_\nu} \bar{E}(j_\pi j_\nu) \hat{n}_{j_\pi} \hat{n}_{j_\nu}$$

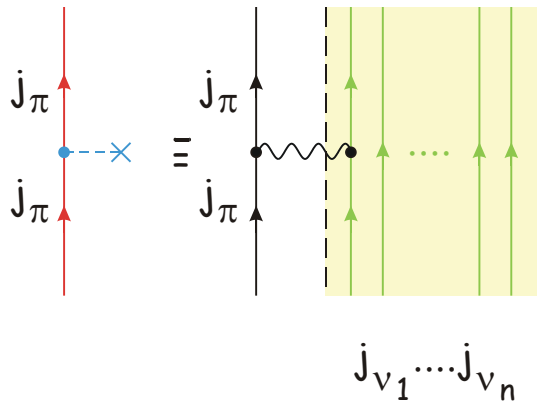
$$\bar{E}(j_\pi j_\nu) = \frac{\sum_J (2J+1) \langle j_\pi j_\nu, J | V_{\pi\nu} | j_\pi j_\nu, J \rangle}{\sum_J (2J+1)}$$

Proton self-energy correction in 'HF+BCS' approximation



$$H = H_M + H_{pair}$$

occupation probabilities
(from BCS)



$$\tilde{\varepsilon}_{j_\pi} = \varepsilon_{j_\pi} + \sum_{j_v} \bar{V}_{j_\pi j_v} (2j_v + 1) v_{j_v}^2$$

average interaction
matrix element

$$\bar{V}_{j_\pi j_v} = \frac{\sum_J \langle j_\pi j_v | V | j_\pi j_v \rangle_J (2J + 1)}{\sum_J (2J + 1)}$$

A.L.Goodman, NPA267, 1 (1977)
R.A.Sorensen, NPA420, 221 (1984)
K.Heyde et al, NPA466, 189 (1987)
A.P.Zuker, NPA576, 65 (1994)

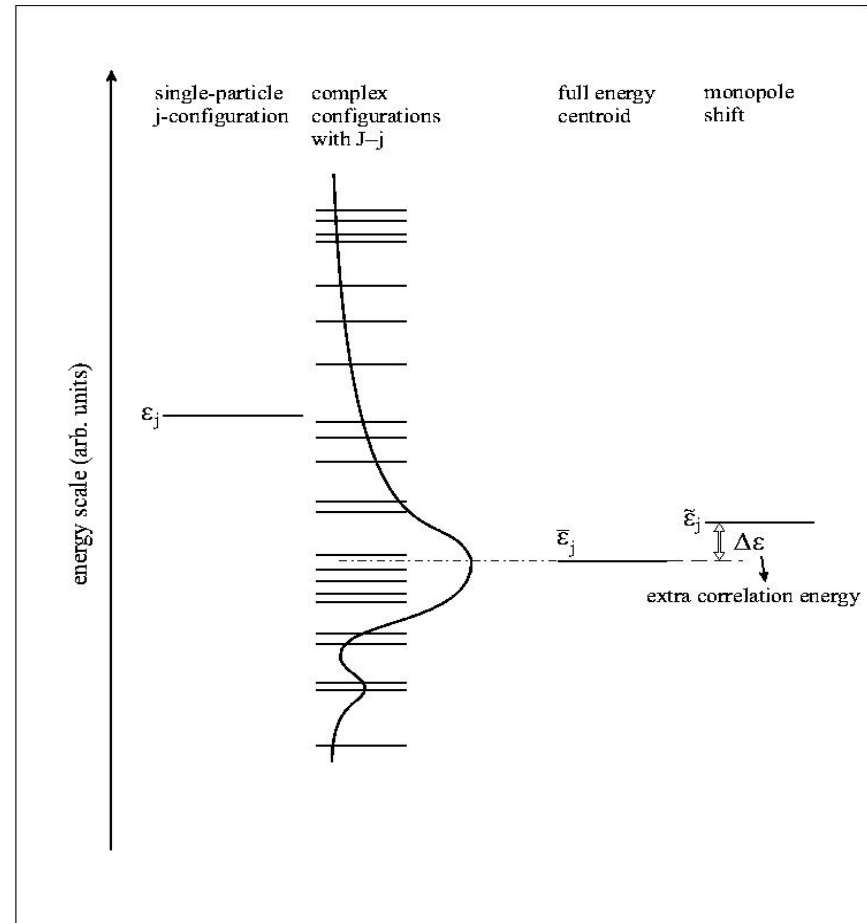
Diagonalization - energy centroid

- nucleus = inert core & valence nucleons

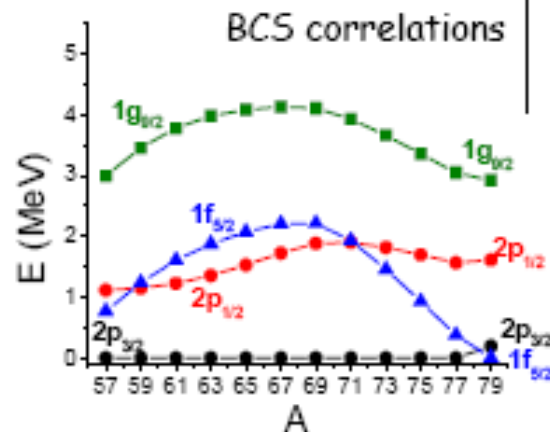
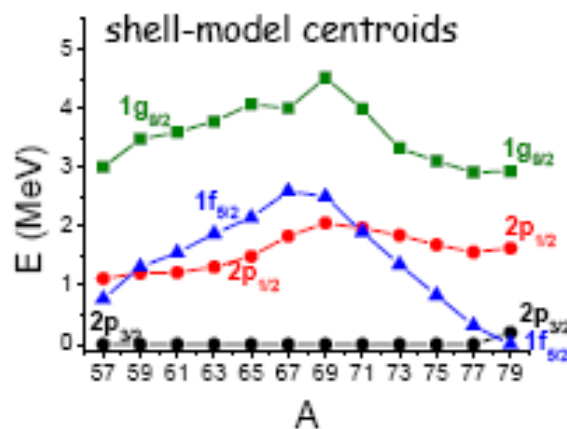
$$H = \sum_{j_\pi} \varepsilon_{j_\pi} \hat{n}_{j_\pi} + \sum_{j_\nu} \varepsilon_{j_\nu} \hat{n}_{j_\nu} + V_{\pi\pi} + V_{\nu\nu} + V_{\pi\nu}$$

- Full shell-model picture
- Construct energy centroid

$$\bar{\varepsilon}_j = \frac{\sum_i E^{(i)}(J) S(J^{(i)}; j)}{\sum_i S(J^{(i)}; j)}$$



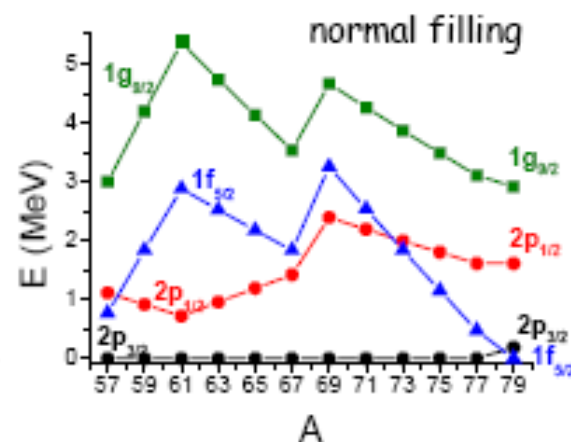
Shell-Model Centroids versus the Monopole Shift in Cu-Isotopes



$$E(nlj) = \frac{\sum_i S_{nlj}^i E_{nlj}^i}{\sum_i S_{nlj}^i}$$

$$\tilde{\epsilon}_{j\pi} = \epsilon_{j\pi} + \sum_{j\nu} \bar{V}_{j\pi j\nu} (2j_\nu + 1) v_{j\nu}^2$$

Microscopic effective interaction from M. Hjorth-Jensen et al, adjusted by F.Nowacki (1996)
ANTOINE shell-model code (E.Caurier)



$$H = H_m(\lambda=0) + H_M(\lambda \neq 0)$$

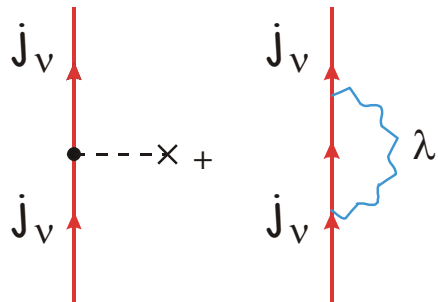
monopole

pairing + higher multipoles

HIGHER MULTIPOLES: coupling to collective excitations of the core (quadrupole, octupole)
Fragmentation

N = 83 nuclei

Interesting case in variation of $1i_{13/2}$ configuraton



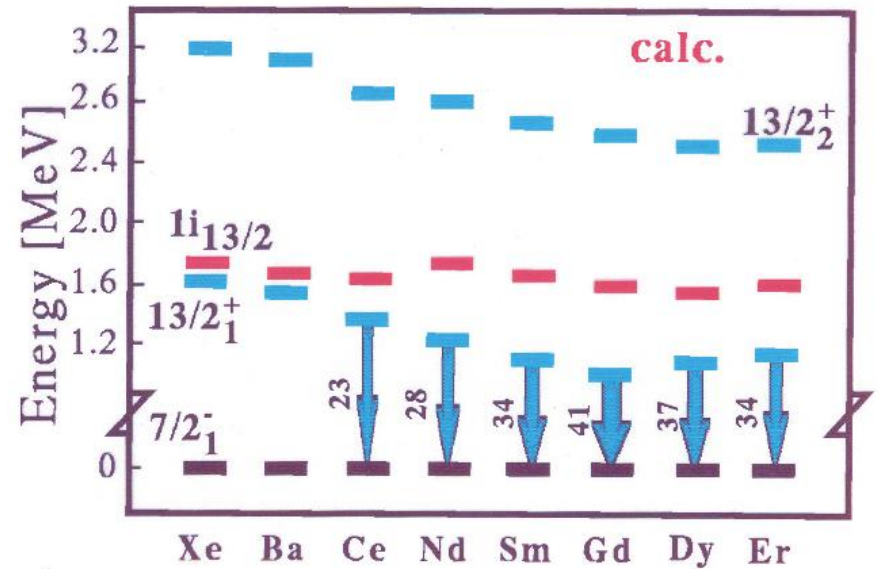
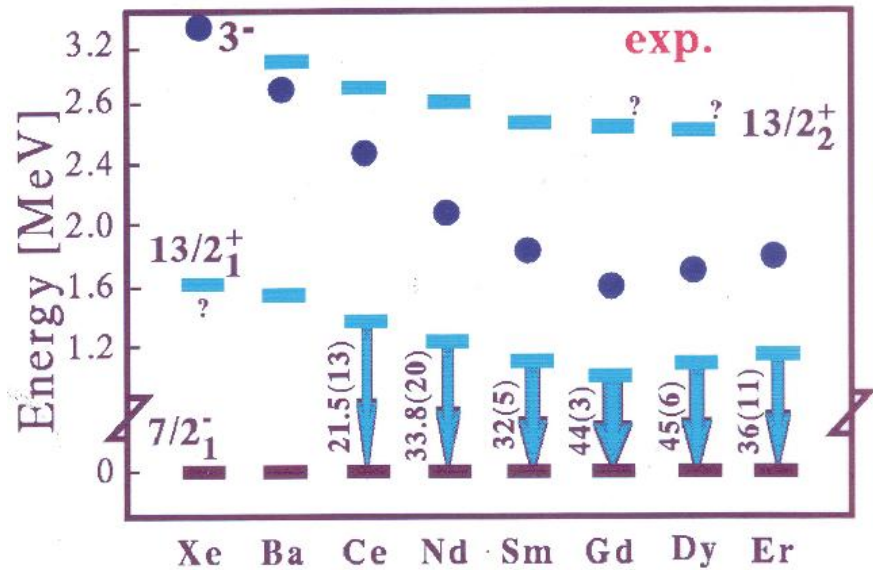
$$\Delta \varepsilon_{j_v} = \sum_{j_{v'}} \frac{\langle j_v | H_{\text{int}} | j_{v'}, 3^-; j_v \rangle \langle j_{v'}, 3^-; j_v | H_{\text{int}} | j_v \rangle}{\varepsilon_{j_v} - (\varepsilon_{j_{v'}} + \hbar \omega_\lambda)}$$

Collective ($\lambda^\pi = 3^-$) isoscalar vibration only moderate overlap with open proton shell. Observable in spectroscopic factors

⇒ need for good data in order to map the variations in ε_{j_v} (and general)

For $13/2_1^+$ $S_{l=6} \simeq 0.5$

The orbital $1i_{13/2}$



Page et al. (JYFL)- N=83 nuclei up to Yb(Z=70), Hf(Z=72), W(Z=74) $7/2^-$, $9/2^-$

Research Topics - present and future projects

K- isotopes from $A=39$ to $A=47$

$N=29$ isotones from Ca to Ni

$N=51$ isotones

Sb isotopes: realistic vs. schematic forces

Bi isotopes

- $N=83$ isotones: work in progress
- Detailed study of decomposition of the nuclear interaction in the central, spin-orbit, tensor,.. channels- comparison with schematic forces and experiment.
- Study of the effects of pairing and coupling to multipole excitations (quadrupole, octupole,..) in the study of fragmentation of single-particle strength. Comparison with large-scale shell-model studies.

Research carried out within the context of the IAP 'Advanced Research on Exotic Nuclei for Nuclear Physics and Nuclear Astrophysics' (BriX) 2007-2011

In close collaboration with:

Smirnova Nadya (CENG-Bordeaux)

Fossion Ruben (UNAM-Mexico)

Van Isacker Piet (GANIL)

Jolie Jan (Univ.Köln)

Many experimentalists at IKS,Leuven, ISOLDE-CERN,
Jyväskylä,GANIL,GSI,..in particular with J.L.Wood (Georgia
Tech., Atlanta)